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The analysis of a transistor cap as a heat dissipator

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THE ANALYSIS OF A TRANSISTOR CAP
AS
A HEAT DISSIPATOR

by

Kathleen Cooper Bryant

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The Analysis of a Transistor Cap as a Heat Dissipator

by

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Lieutenant, United States Navy
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ABSTRACT

This thesis develops and utilizes electro-thermal analogies to analyze the amount of heat dissipated in a typical transistor cap. A series of parametric curves are developed to illustrate the results obtained. These curves may be used by circuit designers in order to obtain a more precise estimate of the heat sink system requirements.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I. INTRODUCTION

A. BACKGROUND

There are at least two important reasons why the junction of a transistor must be kept within prescribed and precise temperature limits. The first of these concerns the reliability of the device and the second relates to the bias or operating point.

It is an established fact that the failure rate of a transistor is a direct function of junction temperature. There is a vast amount of literature that provides curves and tables of semiconductor failure rates as a function of junction temperature. Such a table taken from data provided by Thornell et al [Ref. 1] and Harper [Ref. 2] is shown in Table I . The numbers shown clearly demonstrate the necessity for thermal control to a maximum operating temperature dictated by the requirements for the reliability of commercial and/or military systems.

Because collector current varies directly with junction temperature, it is apparent that, in the collector circuit with fixed bias and external resistance, as in the common emitter configuration, increases in collector current can cause decreases in collector-emitter voltage. A severe decrease of this voltage may shift the bias-stabilization

TABLE I

BASE FAILURE RATES VS. TEMPERATURE

Element	25	50	75	100	125
			Temperature (°C)		
Transistor chips					
Low power	.0001	.0003	.0009	.0027	.007
High power	.0050	.0100	.0300	.0900	.270
Diode chips	.0001	.0003	.0009	.0027	.007
Microcircuits					
Quad gate	.0020	.0036	.0180	.0820	.240
Dual flip-flop	.0040	.0072	.0360	.1640	.480

(Failure rates in percent per thousand hours)

point of an amplifier to a point where the device is operating outside of its specification. This too may be thought of as a reduction of both device and system reliability.

Therefore, in order to obtain optimal performance from a device, the designer must know exactly what his heat dissipating requirements will be. Overestimation and overcompensation in terms of implementing heat dissipating devices can also adversely affect size, cost, and even operation.

B. OBJECTIVES

The goal of this thesis is to provide the designer with curves that will permit a precise estimate of the amount of heat dissipated by the cap of a TO package. A knowledge of the fraction of the heat dissipated by the cap to the total heat dissipated can perhaps lead to a lighter and less costly cooling structure. Although the fraction of the heat dissipated by the cap may be small, any improvement in reducing size and cost may still be sought.

C. PROCEDURE

To obtain the desired curves, this thesis will utilize familiar electrical theory by first developing an electro-thermal analogy for the transistor package. The electro-thermal model will then be analyzed in terms of parameters that equate to those of interest in the transistor package. The result of this analysis will be one unifying procedure that relates the temperature and heat flow at the transistor junction to heat transfer and physical variables such as air flow, temperature levels, and package size. A set of parametric curves will be drawn utilizing this procedure in order to illustrate the relationships that the designer will be required to consider for a particular TO configuration.

II. THE TRANSMISSION LINE ANALOGY FOR THE COOLING FIN

A. THE ELECTRO-THERMAL ANALOG

The flow of electricity and the flow of heat are similar in many ways. Analogies are drawn between the quantities used to describe each process, and electro-thermal analogs in the form of electrical networks are formulated to model and to analyze thermal configurations. The development of these analogies has been covered in depth in several works, including Holman [Ref. 3], Lienhard [Ref. 4], and Incropera and Dewitt [Ref. 5]. A discussion is presented here for purposes of continuity and completeness.

A brief presentation of the heat flow problem in and from a cooling fin is contained in Appendix A, along with a list of assumptions pertaining to the transistor cap that will impact on further derivations. The reader may refer to Appendix A for answers to any questions regarding the heat flow theory considerations necessary to an understanding of what will be proposed in this thesis.

The first step towards development of an electrical model for the transistor cap is the determination of equivalent thermal and electrical quantities. The variables of concern in an electrical network are the voltage or potential, V, and the current, I. The variables of interest

in thermal applications are the temperature excess, Θ, and the heat flow, q. First compare the equations for heat flow by conduction in a rectangular fin and the flow of electricity through an impedance:

$$q = kA\Delta T/\Delta x \tag{2.1}$$

$$I = (1/Z)(\Delta V/\Delta x) \tag{2.2}$$

where k is the thermal conductivity of the fin material, A is the area perpendicular to heat flow, T is the temperature of the fin, and Z is the impedance per unit length of the network. The two equations are of similar form, suggesting that an analogy between the variables can be proposed.

In a second comparison, consider the equations for heat transfer from the surface of the fin by convection and for current through an admittance. Conduction was compared in the foregoing with a series process, because heat flows through the fin against a thermal "resistance". Here, convection may be viewed as a shunt process, because heat is transferred off the surface of the fin into the air or surrounding medium. The two equations are then:

$$\Delta q = hL\Theta\Delta x \tag{2.3}$$

$$\Delta I = YV\Delta x \tag{2.4}$$

where h is the heat transfer coefficient that governs the heat transfer from fin to surroundings, L is the length of the fin, Θ is the temperature excess (the temperature of the fin surface minus the temperature of the surroundings), and Y is the admittance per unit length of the electrical network.

By comparing variables and constants between equations 2.1 and 2.2 and 2.3 and 2.4, the analogies displayed in Table II are easily observed. The equivalent quantities may therefore be used in development of an electrical network model of a thermal configuration, or vice versa.

B. THE TRANSMISSION LINE MODEL

The transistor cap may be divided into two separate parts: the cylindrical body and the circular disk. At this point it is desired to develop an electrical model only for the cylinder. As shown in Figure 2.1, the cylinder may be treated as a rectangular fin with length L, where L is equal to the circumference of the cylinder ($L = \pi d$) and, if r_e is the outer radius of the circular disk, $d = 2r_e$.

In developing the electrical model for the fin, it should be noted from Appendix A that the fin possesses two distributed parameters that, together with its dimensions, can completely determine its thermal behavior: the thermal conductivity, k, and the heat transfer coefficient, h. The behavior of the electrical transmission line is similarly

TABLE II

ELECTRO-THERMAL ANALOGIES

THERMAL QUANTITY		ELECTRICAL QUANTITY
q	(heat flow)	I (current)
Θ	(temperature excess)	V (voltage)
$1/kA$	(thermal "resistance") per unit length	Z (impedance per unit length)
hL	(thermal "admittance") per unit length	Y (admittance per unit length)

dictated by two distributed parameters: the impedance Z per unit length and the admittance Y per unit length. However, as Table II clearly shows, the thermal resistance and the electrical impedance are analogous quantities as are the heat transfer coefficient and the electrical admittance. In addition, the per unit length stipulation pertaining to these quantities demands a consideration of the fin as a distributed configuration. This suggests that the transmission line may be an appropriate model for the fin.

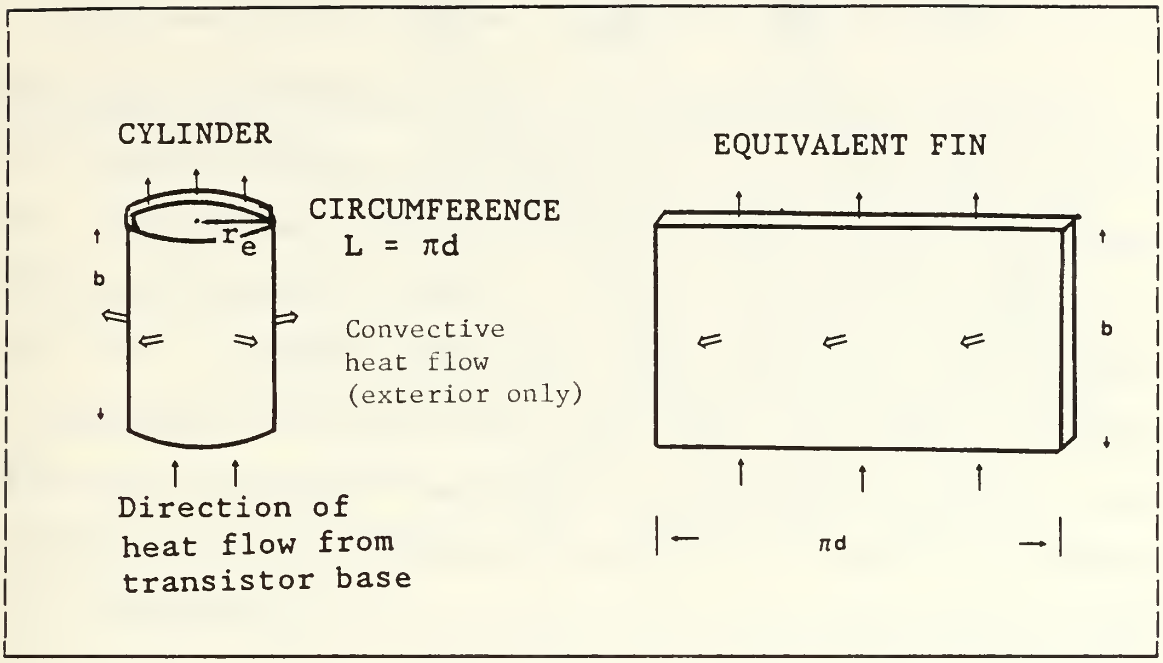

Figure 2.1 The Cylinder as a Rectangular Fin.

Next, consider the well known transmission line equations [Ref. 6: page 215]:

$$d^2V/dx^2 - \gamma^2 V = 0 \tag{2.5}$$

$$d^2I/dx^2 - \gamma^2 I = 0 \tag{2.6}$$

where $\gamma = \sqrt{ZY}$. These equations may be compared with equation A.10 for temperature excess in the fin:

$$d\Theta^2 - m^2\Theta = 0 \tag{2.7}$$

where $m^2 = h/kA$ (for heat transfer off one side of the cylinder only). Consideration of the quantities in Table II indicates that:

$$\gamma = \sqrt{ZY} = \sqrt{\frac{hL}{kA}} = \sqrt{\frac{hL}{k\delta L}} = \sqrt{\frac{h}{k\delta}} = m \qquad (2.8)$$

Since m^2 is equivalent to γ^2, the differential equations of equations 2.5 and 2.7 are analogous in terms of both variables and constants. Because they are identical in form, they will possess identical general solutions, and if evaluated for correspondingly equivalent boundary conditions, they will have identical particular solutions. The conclusion, therefore, is that the transmission line may be used as a model in analyzing the behavior of the cooling fin in general and the side of the TO can in particular.

C. PROPERTIES OF THE TRANSMISSION LINE

Before proceeding further, it may be useful to discuss some finer points pertaining to the transmission line and their impact on application of the line as a model for the cooling fin.

Usually the series impedance Z is defined in terms of the resistance and inductance of the line, as in:

$$Z = R + j\omega L \qquad (2.9)$$

The shunt admittance Y is usually defined in terms of the conductance and capacitance:

$$Y = G + j\omega C \qquad (2.10)$$

The concept of inductance, however, has no thermal equivalent because no thermal element exists to "store" temperature excess. Additionally, in this application, no heat sources or sinks are allowed to exist in the fin, so that no heat can be stored. This implies that no term equivalent to capacitance is present. Moreover, thermal capacitance is a phenomenon that relates to transient heat flow, and the analysis contained in this thesis deals strictly with the steady state. Equations 2.9 and 2.10 therefore reduce to:

$$Z = R \tag{2.11}$$

$$Y = G \tag{2.12}$$

The transmission line analogy then reduces to the particular case of the resistive-conductive (R-G) or "lossy" transmission line.

Several other parameters are of interest in the description of a transmission line. The characteristic impedance of the line, Z_0, is defined to be $\sqrt{Z/Y}$. Conversely, the characteristic admittance of the line, Y_0, is $1/Z_0$, or $\sqrt{Y/Z}$. From Table II, the equivalent thermal "characteristic impedance" and "characteristic admittance" can be calculated to be:

$$Z_0 = 1/(L\sqrt{hk\delta}) \tag{2.13}$$

$$Y_0 = L\sqrt{hk\delta} \tag{2.14}$$

respectively. Note that this expression for Y_0 corresponds to the definition of thermal "characteristic admittance" proposed in Appendix A.

A term that was used earlier, γ, is designated as the propagation constant and is usually of the form

$$\gamma = \alpha + j\beta \tag{2.15}$$

where α is the attenuation constant and β is the phase shift constant. Since phase shifts are meaningless in the thermal process (i.e. the quantity γ is real so that $\beta = 0$), the electrical model may assume that the propagation constant is equal to the attenuation, i.e.

$$\gamma = \alpha \tag{2.16}$$

Because $\gamma = \alpha = \sqrt{ZY}$, the equivalent thermal attenuation factor is m, where $m = \sqrt{h/k\delta}$. This, too, corresponds to the suggestion provided in Appendix A that m can be considered as the fin "attenuation" factor. These points are summarized in Table III .

TABLE III

TRANSMISSION CHARACTERISTIC ANALOGIES

ELECTRICAL		THERMAL
$\sqrt{Z/Y}$	characteristic impedance	$1/(L\sqrt{hk\delta})$
$\sqrt{Y/Z}$	characteristic admittance	$L\sqrt{hk\delta}$
$\sqrt{ZY}$	attenuation constant	$\sqrt{h/k\delta}$

D. THE FIN AS A TWO-PORT NETWORK

1. The Electrical Two-Port Network

It is known from electrical theory that the transmission line may be represented by a two-port network. The representation of a two-port is shown in Figure 2.2. Note that the arrows indicate the conventional direction of positive current, which is always flowing into the network from either port.

Analysis of a two-port network concerns only the conditions at the two ports. Conditions inside the network are usually neither desired nor available. For the transmission line, the conditions at the ports are the voltage and current at the "sending" end, and the voltage

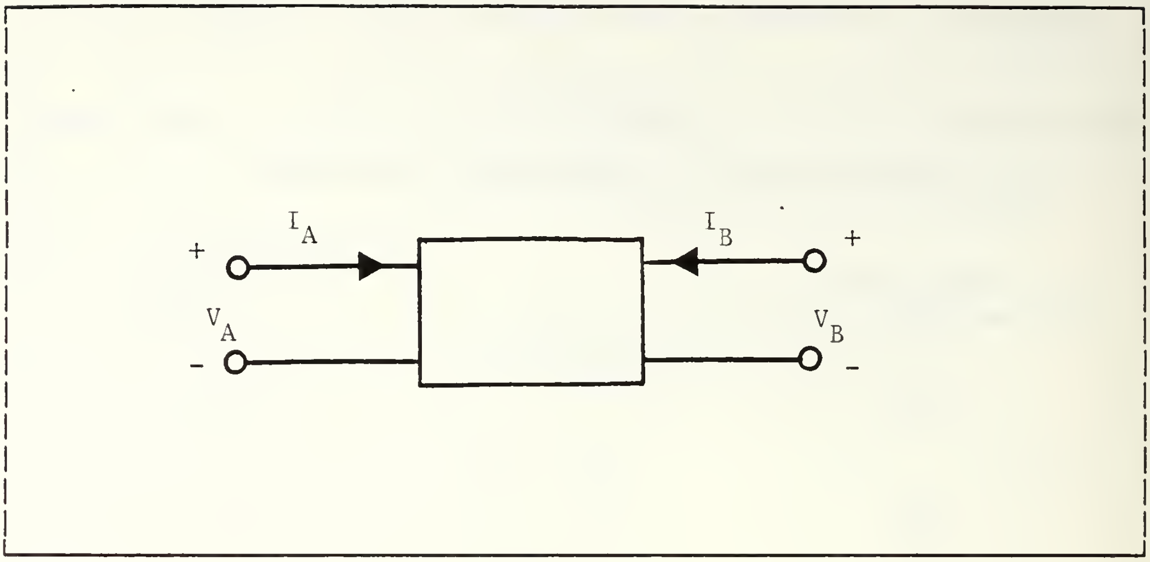

Figure 2.2 A Two-Port Network.

and current at the "receiving" end. Any two of these four variables may be represented in a two-port by the linear superposition of the effects of the other two variables, such as:

$$V_S = AV_R - BI_R \tag{2.17}$$

$$I_S = CV_R - DI_R \tag{2.18}$$

where V_S and I_S are the voltage and current at the sending end, V_R and I_R are the voltage and current at the receiving end, and where the minus signs are required because convention has the current I_R leaving port B. This pair of equations is often represented by the transmission parameterization:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ -I_R \end{bmatrix} \quad (2.19)$$

In this particular case, the conditions at one port (the sending end) have been selected as the dependent variables. They are induced by the values of the independent variables, which in this case are the conditions at the receiving end. The linear transformation matrix which maps receiving end conditions to sending end conditions is known as the transmission parameter or ABCD matrix.

The transmission parameter formulation is germane to the application covered in this thesis because it allows the cap or disk of the TO can to be connected to the side of the can in cascade. It may be recalled that when two or more two-ports are connected in cascade, the entire configuration may be expressed by an equivalent transmission parameter matrix that is equal to the matrix product of the individual transmission parameter matrices. The cascade connection for the electrical case is displayed in Figure 2.3.
Here, with regard to Figure 2.3,

$$\begin{bmatrix} A_{eq} & B_{eq} \\ C_{eq} & D_{eq} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

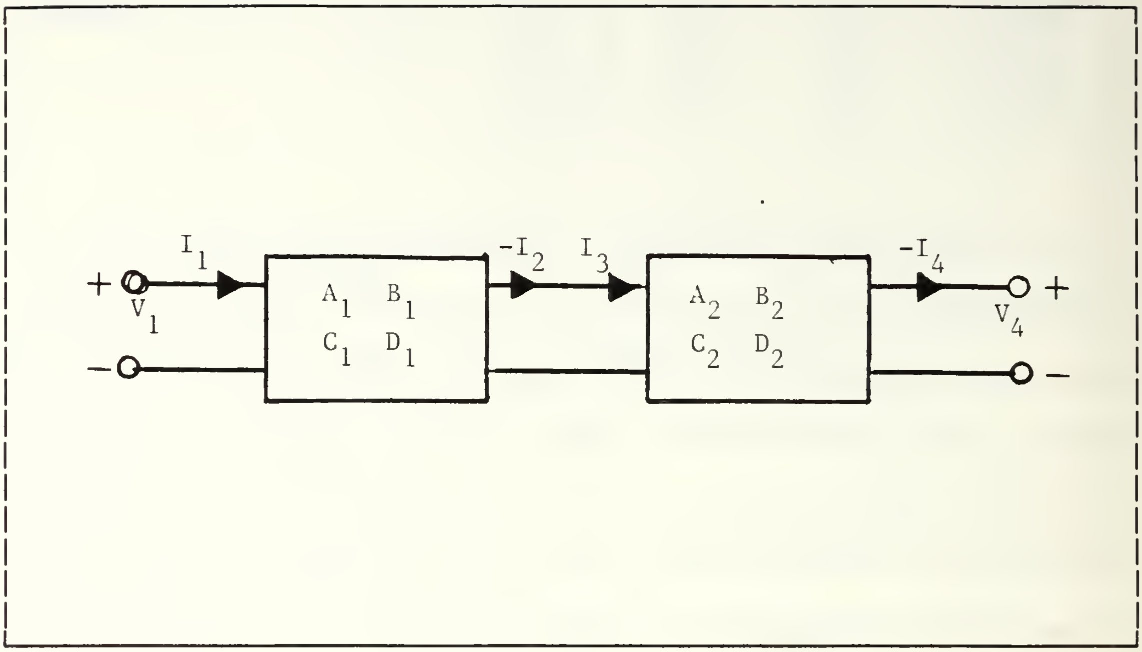

Figure 2.3 Cascaded Two-Ports.

Because it has been shown that the transmission line is an acceptable model for the fin, and the transmission line is known to have a two-port representation with a transmission parameter matrix, it is proposed that an equivalent thermal two-port and transmission parameter matrix representation must exist.

2. <u>The Thermal Two-Port</u>

The conventional representation of the transmission line as a two-port was shown in Figure 2.2. Based on the analogies listed in Table II, the proposed thermal two-port for the case of the fin dissipating heat by convection to the environment is illustrated in Figure 2.4. Note that

here the conventional positive direction of heat flow at the "tip end" is opposite to that of the receiving end current in the electrical two-port. This must be represented in this manner because the limiting assumptions (listed in Appendix A) dictate heat flow in one direction only. This heat flow, in a cooling application, is from the base of the fin to the tip.

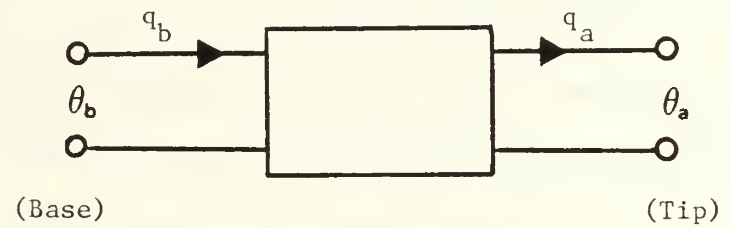

Figure 2.4 Proposed Thermal Two-Port.

Based on Figure 2.4, the equivalent transmission parameter matrix representation would be:

$$\begin{bmatrix} \Theta_b \\ q_b \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \Theta_a \\ q_a \end{bmatrix}$$

where it can be observed that the minus sign present in the electrical representation has been deleted due to the direction of heat flow.

To find the transmission parameters for the fin, the heat flow equation for temperature excess can be solved using initial value data. Appendix A provides the differential equation for the fin temperature excess:

$$d\Theta^2 - m^2\Theta = 0 \tag{2.20}$$

where $m = \sqrt{h/k\delta}$ for convective heat flow off one side only. This equation has a general solution:

$$\Theta(x) = C_1e^{mx} + C_2e^{-mx} \tag{2.21}$$

The initial conditions at the base are:

$$\Theta(x{=}b) = \Theta_b \tag{2.22}$$

$$q(x{=}b) = q_b \tag{2.23}$$

When equation 2.22 is substituted into equation 2.21 above, the result is:

$$\Theta_b = C_1e^{mb} + C_2e^{-mb} \tag{2.24}$$

and from the Fourier Law of Heat Conduction:

$$q(x) = kA\ d\Theta/dx = kAm[\ C_1e^{mx} + C_2e^{-mx}\ ] \tag{2.25}$$

the substitution of equation 2.23 into equation 2.24 yields:

$$q_b = kAm\left[C_1 e^{mb} - C_2 e^{-mb} \right] \qquad (2.26)$$

Equations 2.24 and 2.26 are two equations in two unknowns (C_1 and C_2), and these can be solved simultaneously to determine that:

$$C_1 = e^{-mb}\left(\frac{\Theta_b}{2} + \frac{Z_0 q_b}{2}\right) \qquad (2.27)$$

$$C_2 = e^{mb}\left(\frac{\Theta_b}{2} + \frac{Z_0 q_h}{2}\right) \qquad (2.28)$$

where Z_0 is the characteristic impedance discussed in section C of this chapter.

Use of these values of the constants C_1 and C_2 provides the particular solution to equation 2.20:

$$\Theta(x) = .5\left[\left(\Theta_b e^{-mb} + Z_0 q_b e^{-mb}\right)e^{mx} + \left(\Theta_b e^{mb} - Z_0 q_b e^{mb}\right)e^{-mx}\right]$$

and when rearranged to show a superposition of the effects of Θ_b and q_b this becomes:

$$\Theta(x) = .5\left[\Theta_b\left(e^{m(b-x)} + e^{-m(b-x)}\right) - Z_0 q_b\left(e^{m(b-x)} - e^{-m(b-x)}\right)\right] \qquad (2.30)$$

This may alternately be expressed in terms of hyperbolic sines and cosines as:

$$\Theta(x) = \Theta_b \cosh m(b-x) - Z_0 q_b \sinh m(b-x) \qquad (2.31)$$

It is then easy to obtain the heat flow, $q(X)$, through an application of the Fourier Law:

$$q(x) = -Y_0 \sinh m(b-x) + q_b \cosh m(b-x) \qquad (2.32)$$

where $Y_0 = L\sqrt{hk\delta}$ is the characteristic thermal admittance of the fin.

It is necessary to evaluate equations 2.31 and 2.32 at x=0 in order to obtain a relationship between tip and base conditions of temperature excess and heat flow. Under this particular circumstance:

$$\Theta_a = \Theta_b \cosh(mb) - Z_0 q_b \sinh(mb) \qquad (2.33)$$

$$q_a = -Y_0 \sinh(mb) + q_b \cosh(mb) \qquad (2.34)$$

and these equations can be put into matrix form:

$$\begin{bmatrix} \Theta_a \\ q_a \end{bmatrix} = \begin{bmatrix} \cosh(mb) & -Z_0 \sinh(mb) \\ -Y_0 \sinh(mb) & \cosh(mb) \end{bmatrix} \begin{bmatrix} \Theta_b \\ q_b \end{bmatrix}$$

This is the inverse thermal transmission representation.

To obtain the transmission parameter matrix, it is required to show the base conditions as dependent on tip conditions, not vice versa as depicted in equations 2.33 and

2.34. The necessary rearrangement is easily accomplished by taking the inverse and the result is such that:

$$\begin{bmatrix} \Theta_b \\ q_b \end{bmatrix} = \begin{bmatrix} \cosh(mb) & Z_0 \sinh(mb) \\ Y_0 \sinh(mb) & \cosh(mb) \end{bmatrix} \begin{bmatrix} \Theta_a \\ q_a \end{bmatrix} \quad (2.35)$$

From this one can see that the base conditions are dependent upon the linear superposition of the effects of the tip conditions. This final form is the equivalent transmission parameter matrix for the fin.

Presented below is the known transmission parameter matrix for the transmission line [Ref. 6: page 217]. In a comparison of the matrix relationships for the thermal two-port and the electrical two-port, one finds the expected similarity.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma b) & Z_0 \sinh(\gamma b) \\ Y_0 \sinh(\gamma b) & \cosh(\gamma b) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

From this similarity it is concluded that the fin has a thermal two-port representation and a transmission parameter matrix as indicated by equation 2.35.

E. THE CASE OF A TWO-PORT CASCADED WITH A SHUNT ADMITTANCE

As already indicated, two-port theory states that the equivalent transmission parameter matrix for two or more two-port networks connected in cascade is equal to the product of the individual two-port matrices. In the particular case where a two-port network is connected in cascade with a shunt admittance, as illustrated in Figure 2.5, the equivalent two-port transmission parameter matrix is even easier to obtain. No calculation of a transmission parameter matrix for the shunt admittance is necessary, although it is easily derived. The transmission parameter matrix for the shunt admittance is known to be of the form [Ref. 7: p. 163]:

$$\begin{bmatrix} 1 & 0 \\ 0 & Y_{in} \end{bmatrix}$$

When this equivalent shunt transmission parameter matrix is premultiplied by the transmission parameter matrix that represents the rest of the network, the result is the equivalent transmission parameter matrix for the entire network. In this case the equivalent transmission parameter matrix is of the form shown below:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{eq} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_{in} & 1 \end{bmatrix} = \begin{bmatrix} A+BY_{in} & B \\ C+DY_{in} & D \end{bmatrix} \quad (2.36)$$

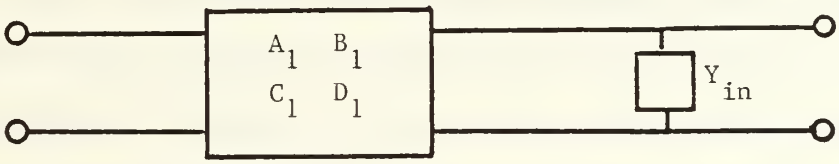

Figure 2.5 A Two-Port Cascaded With a Shunt Admittance.

This reasoning can also be extended to the performance of the fin which may be discussed using two-port terminology. Consequently, if an equivalent "input admittance" can be determined for the top disk of the transistor cap, the entire cap (the cylindrical sides which are connected in cascade to the circular disk) may be modeled by one two-port network by using the relationship shown in equation 2.36.

The next chapter will concentrate on developing an equivalent "input admittance" for the circular disk.

F. CAUTIONS PERTAINING TO THE USE OF THERMAL TWO-PORT ANALOGIES

The purpose of this chapter was to develop a thermal two-port representation for the rectangular cooling fin. This was done through the use of electro-thermal analogs, and the similarity between the thermal two-port and electrical two-port was duly noted. However, significant differences exist between the two and these differences must be taken into consideration when attempting to use electrical two-port theory in thermal applications. These differences are:

1) In the electrical two-port either port can be considered as the input port. In the thermal two-port for the fin, the base is always considered as the input port.

2) The thermal two-port always has a height-coordinate associated with it. It is measured in a direction taken positive from the tip to the base of the fin. The electrical two-port has no equivalent directional dependency.

3) The direction of the heat flow at the output port (the tip of the fin) in the thermal two-port is opposite that of the current flow at the output port of an electrical two-port. This difference has a tremendous effect on conversions between the two-port parameters.

III. THE INPUT ADMITTANCE OF A CIRCULAR DISK

A. INTRODUCTION

The objective of this chapter is to develop an expression for the "input admittance", Y_{in}, of the disk, which is the admittance seen looking into the disk. Physically, it relates to the ease in which the heat will flow into the disk, and here the entire disk may be replaced in the electro-thermal analog by the input admittance alone.

Generally, admittance is expressed as the ratio of current to voltage (I/V). Using the electro-thermal analog, the equivalent thermal admittance is defined to be the ratio of heat flow to the temperature excess (q/Θ). The input admittance in this application is specifically the admittance at the edge of the disk, and this will be denoted as .

B. THE GENERAL EQUATIONS FOR THE DISK

To determine the input admittance of the disk, the differential equations of heat flow in the disk must be solved. To obtain the differential equations, first consider the differential element between radius r and $r + \Delta r$ in the disk as shown in Figure 3.1. Notice that the positive orientation of the radial coordinate is from the center to the edge of the disk and that heat enters the

element at radius $r + \Delta r$ and leaves the element at radius r by conduction. Additional heat will be lost through the top face of the element by convection.

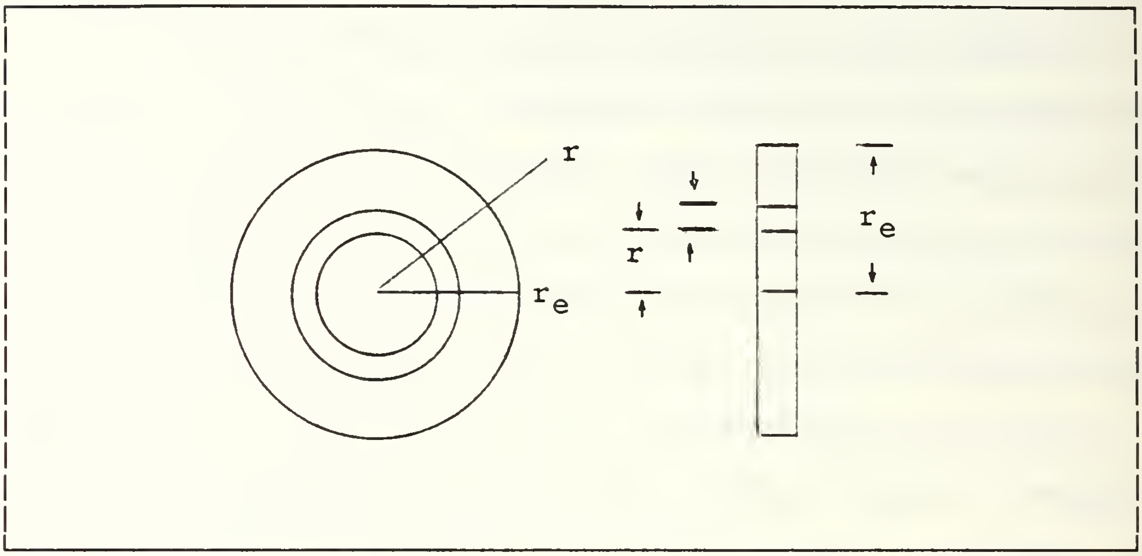

Figure 3.1 Diagram of Circular Disk.

The heat entering the element by conduction at radius $r + \Delta r$ will be:

$$q(r+\Delta r) = kAdT/dr\Big|_{r+\Delta r} = kAd\Theta/dr\Big|_{r+\Delta r} = 2\pi r\delta kd\Theta/dr\Big|_{r+\Delta r} \quad (3.1)$$

where Θ is used instead of T because it has been postulated that $\Theta = T - T_a$ so that:

$$d\Theta = dT \quad (3.2)$$

Similarly, the heat leaving the element by conduction at radius r is equal to:

$$q_r = kA\ dT/dr = 2\pi rk\delta\ d\Theta/dr \quad (3.3)$$

and the heat lost through the top surface by convection must equal:

$$q_s = h_d \Theta \Delta S = h_d \Theta (2\pi r \Delta r - \pi \Delta r^2) \qquad (3.4)$$

An energy balance (the law of conservation of energy) indicates that, in the steady state, the total heat entering the element must equal the total heat leaving the element:

$$q_{r+\Delta r} = q_r + q_s \qquad (3.5)$$

and use of equations 3.1, 3.3, and 3.4 in equation 3.5 yields:

$$2\pi r \delta k \; d\Theta/dr = 2\pi r \delta k \; d\Theta/dr + h_d \Theta (2\pi r \Delta r - \pi \Delta r^2) \qquad (3.6)$$

A rearrangement of equation 3.6 yields:

$$\frac{r \; d\Theta/dr\Big|_{r+\Delta r} - r \; d\Theta/dr\Big|_{r}}{\Delta r} = \frac{h\Theta r(1 - \Delta r/2)}{k\delta} \qquad (3.7)$$

and in taking the limit as $r \to 0$, one obtains:

$$d/dr \; (r \; d\Theta/dr) - (h_d/k\delta) r\Theta = 0 \qquad (3.8)$$

or

$$r(d^2\Theta/dr^2) + d\Theta/dr - n^2 r \Theta = 0 \qquad (3.9)$$

Here $n = \sqrt{h_d/k\delta}$, where h_d is the heat transfer coefficient for the disk.

Multiplication of equation 3.9 throughout by r puts it in the form of a modified Bessel equation:

$$r^2 d\Theta^2/dr^2 + r\, d\Theta/dr - n^2 r^2 \Theta = 0 \tag{3.10}$$

and the general solution is: [Ref. 8: p. 417]:

$$\Theta(r) = C_1 I_0(nr) + C_2 K_0(nr) \tag{3.11}$$

where C_1 and C_2 are constants to be determined from the appropriate boundary conditions.

C. APPLICATION OF THE BOUNDARY CONDITIONS

One boundary condition for the circular disk may be obtained by considering the temperature excess at the edge ($r = r_e$) as a kn- quantity:

$$\Theta(r=r_e) = \Theta_e \tag{3.12}$$

However, an inspection of equation 3.11 shows that at r=0, $\Theta(r=0)$ will be unbounded because $K_0(0)$ is unbounded. Therefore, in order to keep $\Theta(r=0)$ finite, C_2 must equal zero. Thus, with $C_2 = 0$, equation 3.11 can then be restated as:

$$\Theta(r) = C_1 I_0(nr) \tag{3.13}$$

Then substitution of the boundary condition of equation 3.12 into equation 3.13 yields:

$$\Theta(r=r_e) = \Theta_e = C_1 I_0(nr_e) \tag{3.14}$$

The constant C_1 may therefore be represented as:

$$C_1 = \Theta_e / I_0(nr_e) \tag{3.15}$$

and substitution of this result into equation 3.13 provides the temperature excess of the disk as a function of the radial coordinate r:

$$\Theta(r) = \Theta_e[I_0(nr)/I_0(nr_e)] \tag{3.16}$$

The heat flow may then be obtained by once again employing Fourier's Law:

$$q = kA\, d\Theta/dr \tag{3.17}$$

hat, with $\Theta(r)$

o that, with (r) from equation 3.16:

$$q(\quad 2\pi r k\delta\, [\; n\Theta_e \tag{3.18}$$

The expression for the thermal input admittance may be obtained from equations 3.16 and 3.18 as shown:

$$Y_{in} = q(r_e)/\Theta(r_e) = Y_0 I_1(nr_e)/I_0(nr_e) \tag{3.19}$$

where $Y_0 = 2\pi r_e \sqrt{h_d k\delta}$ is the characteristic admittance of the disk as discussed in Chapter I.

This thermal input admittance, q_e/Θ_e, will therefore be used as a single element admittance to represent the

circular disk in further analysis of the entire transistor cap.

IV. DETERMINATION OF THE INPUT ADMITTANCE OF THE CAP

Results from the previous two chapters can now be combined to achieve the goal of determining the input admittance (Y_{in}) at the base of the transistor cap with respect to the parameters involved, including the quantity of air flow, dimensions of the cap, and materials used in the cap.

In Chapter I, it was stated that because a two port representation had been determined for the cylindrical side of the TO can, determination of the input admittance of the circular disk was the only quantity needed to obtain Y_{in} for the entire cap, as long as the disk could be considered as being connected in cascade with the cylinder.

To illustrate that the disk is indeed connected in cascade with the cylinder, consider again the conventional two port cascade representation displayed in Figure 4.1.

From this diagram it can be seen by continuity that q_2 must equal q_3 and by compatibility Θ_2 must equal Θ_3. The physical interpretation is that the heat flowing out of the cylinder must equal that flowing into the disk, and that the temperature excess at the top of the cylinder must match that at the edge of the disk.

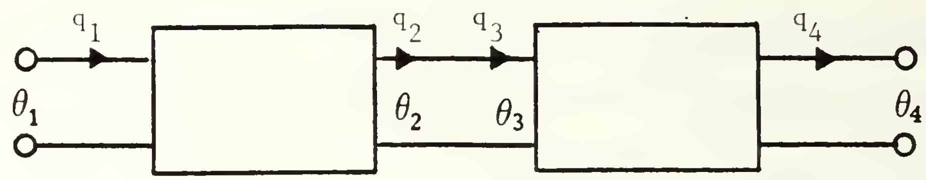

Figure 4.1 Cascaded Two-Port Networks.

These conditions are guaranteed through the limiting assumptions that the cap is constructed of the same material throughout with uniform thickness. Because there is no surface area at the point of interest, no heat will be lost via convection. Thus all of the heat flowing out of the cylinder must be entering the disk. This indicates further that the temperature excess will not change between the top of the cylinder and the edge of the disk. Of course, intuition provides the fact that there can be no temperature discontinuity.

The electrical network model for the cap can therefore be represented as a transmission line terminated in a shunt admittance. Using the two port representation of the transmission line, the network is shown in Figure 4.2.

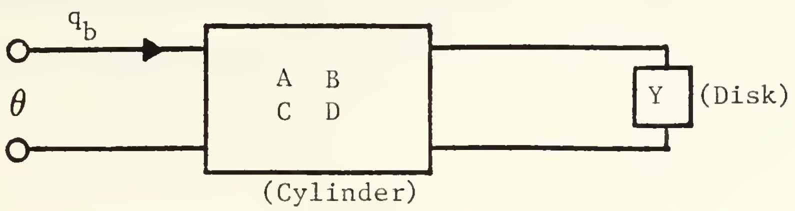

Figure 4.2 The Electrical Model for the Transistor Cap.

It has also been shown that for a two port with a given transmission parameter matrix terminated by a shunt admittance, the equivalent two port representation is:

$$\begin{bmatrix} \Theta_b \\ q_b \end{bmatrix} = \begin{bmatrix} A+BY_{in} & B \\ C+DY_{in} & D \end{bmatrix} \begin{bmatrix} \Theta_c \\ q_c \end{bmatrix} \qquad (4.1)$$

where Θ_c and q_c are respectively the temperature excess and heat flow at the center of the disk.

However, as was noted earlier, no temperature gradient can exist at the center of the disk, thus $q_c = 0$ and this fact can be used to easily show that an expansion of equation 4.1 provides:

$$\Theta_b = (A + BY_{in})\ \Theta_c \tag{4.2}$$

$$q_b = (C + DY_{in})\ \Theta_c \tag{4.3}$$

To obtain the desired input admittance at the base, namely q_b/Θ_b, the ratio of equations 4.2 and 4.3 can be taken, to yield:

$$q_b/\Theta_b = (C + DY_{in})/(A + BY_{in}) \tag{4.4}$$

Then, using the appropriate substitutions:

$$A = \cosh(mb)$$

$$B = Z_{0s}\sinh(mb)$$

$$C = Y_{0s}\sinh(mb)$$

$$D = \cosh(mb)$$

$$Y_{in} = Y_{0d}[I_1(nr_e)/I_0(nr_e)]$$

the relationship for the input admittance at the base of the transistor cap is:

$$Y_{in} = \frac{Y_{0s}\sinh(mb) + Y_{0d}\cosh(mb)\ [I_1(nr_e)/I_0(nr_e)]}{\cosh(mb) + Z_{0s}Y_{0d}\sinh(mb)\ [I_1(nr_e)/I_0(nr_e)]} \tag{4.5}$$

where $Y_{0d} = 2\pi r_e \sqrt{h_d k\delta}$ and $Y_{0s} = 2\pi r_e \sqrt{h_s k\delta}$, $m = \sqrt{h_s/k\delta}$ and $n = \sqrt{h_d/k\delta}$, given that h_s is the heat transfer coefficient of the cylindrical sides and h_d is the heat transfer coefficient of the circular disk.

V. DISCUSSION OF PERFORMANCE CURVES

A series of parametric curves illustrating the relationship between the input admittance of the cap as given by equation 4.5 as a function of the cap dimensions and the heat transfer parameters h and k is displayed in alternate forms in Figures 5.2, 5.3, and 5.4. This chapter will discuss the development of these curves, and the possible ways that they might be utilized.

A. DEVELOPMENT OF CURVES

The TO3 package was chosen as the configuration on which to base the illustrations. Dimensions of this package are shown in Figure 5.1, taken from the D.A.T.A. handbook [Ref. 9]. Where Figure 5.1 indicates a range of lengths or heights, the largest value was used. A standard thickness of 0.03 inches, or 0.0762 cm, was also used.

After specifying the package size, the procedure for obtaining the data for the curves in Figure 5.2 was as follows: For each inlet air temperature, an air velocity was assumed. Then the corresponding heat transfer coefficients for the top and sides were determined. The heat transfer coefficient for the top was obtained through the use of the McGraw-Hill software package "Heat Transfer Software" [Ref. 10], designed for the IBM PC. The heat

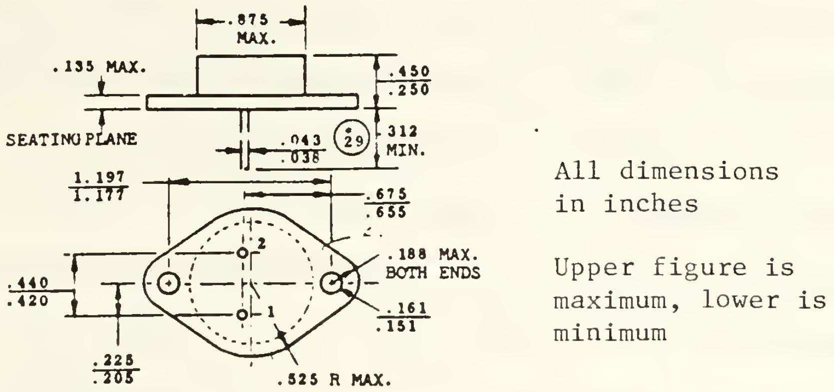

Figure 5.1 Outline Drawing of a TO3 Package.

transfer coefficient for the side was calculated from Hilpert's correlation [Ref. 11]:

$$h = (k/d)\ C\ (u_\infty d/\nu_f)^n\ Pr^{1/3} \qquad (5.1)$$

where u_∞ is the free stream velocity, Pr is the Prandtl number, d is the diameter of the cylinder, k_f is the thermal conductivity of the fluid, ν_f is the kinematic viscosity of the fluid, and C and n are constants that depend on the value of the Reynolds number term $(u_\infty d/\nu_f)$. The heat transfer coefficients used in determining the curves presented here can be found in Appendix B.

A computer program (found in Appendix C) was then used to calculate the resulting input admittance as defined by equation 4.5. This entire procedure was then repeated for other inlet air velocities and temperatures.

The procedure for obtaining data for Figures 5.3 and 5.4 was similar, except that in these cases an inlet air temperature of 25 C was used in all calculations, and the height of the cap (Figure 5.3) or the thickness (Figure 5.4) was taken as the variable parameter.

B. USE OF CURVES

A sample problem may best serve to illustrate how these curves may be utilized. For example, consider a situation where 40 W must be dissipated in an air stream that is flowing at 5 m/s at 50 C. The junction to case resistance is 0.4 C/W, and the temperature at the junction is 125 C.

The case temperature will be 109 C, because:

$$T_c = T_j - 40\,(0.4) = 125 - 16 = 109\ ^{\circ}C \qquad (5.2)$$

The temperature excess may then be calculated to be:

$$\Theta_b = T_c - T_s = 109 - 50 = 59\ ^{\circ}C \qquad (5.3)$$

where T is the free stream temperature. The input admittance, Y , is obtained from Figure 5.2 (for an inlet air temperature of 50 C and a velocity of 5 m/s) and is approximately 0.052 W/ C. From this value of Y , q (which

is the heat dissipation of the cap) may be calculated by observing that:

$$Y_{in} = q_b/\Theta_b \tag{5.4}$$

or

$$q_b = Y_{in}\,\Theta_b \tag{5.5}$$

Hence, in this example,

$$q_b = (0.052)(59) = 3.068\ \mathrm{W} \tag{5.6}$$

The cap dissipates 3.068 watts out of the total dissipation of 40 watts. This amounts to approximately 7.5% of the total dissipation. The designer now has a more precise value of the heat dissipation to be applied to the heat sinking system.

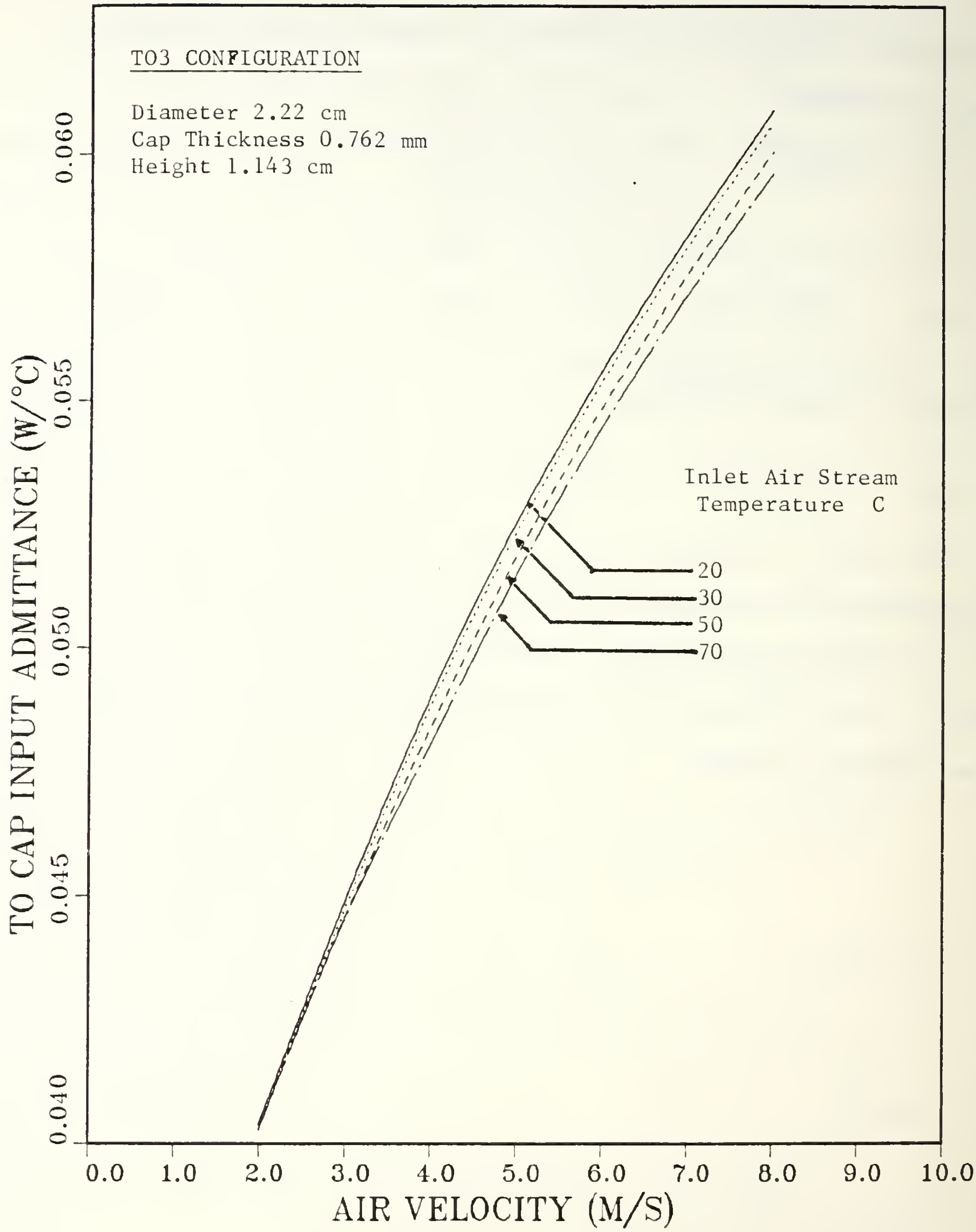

Figure 5.2 Input Admittance vs. Air Velocity with Varying Air Temperatures.

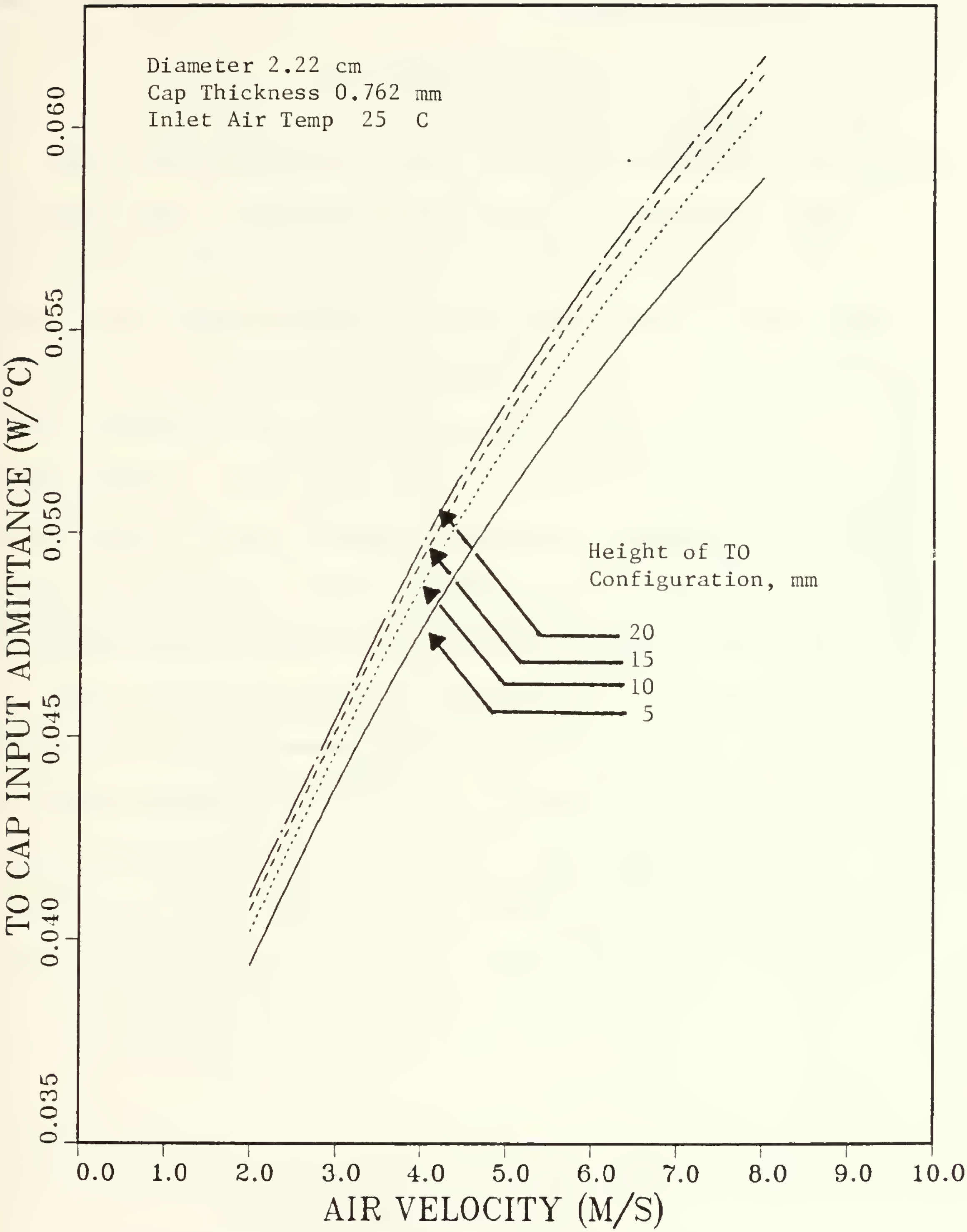

Figure 5.3 Input Admittance vs. Air Velocity with Varying Package Heights.

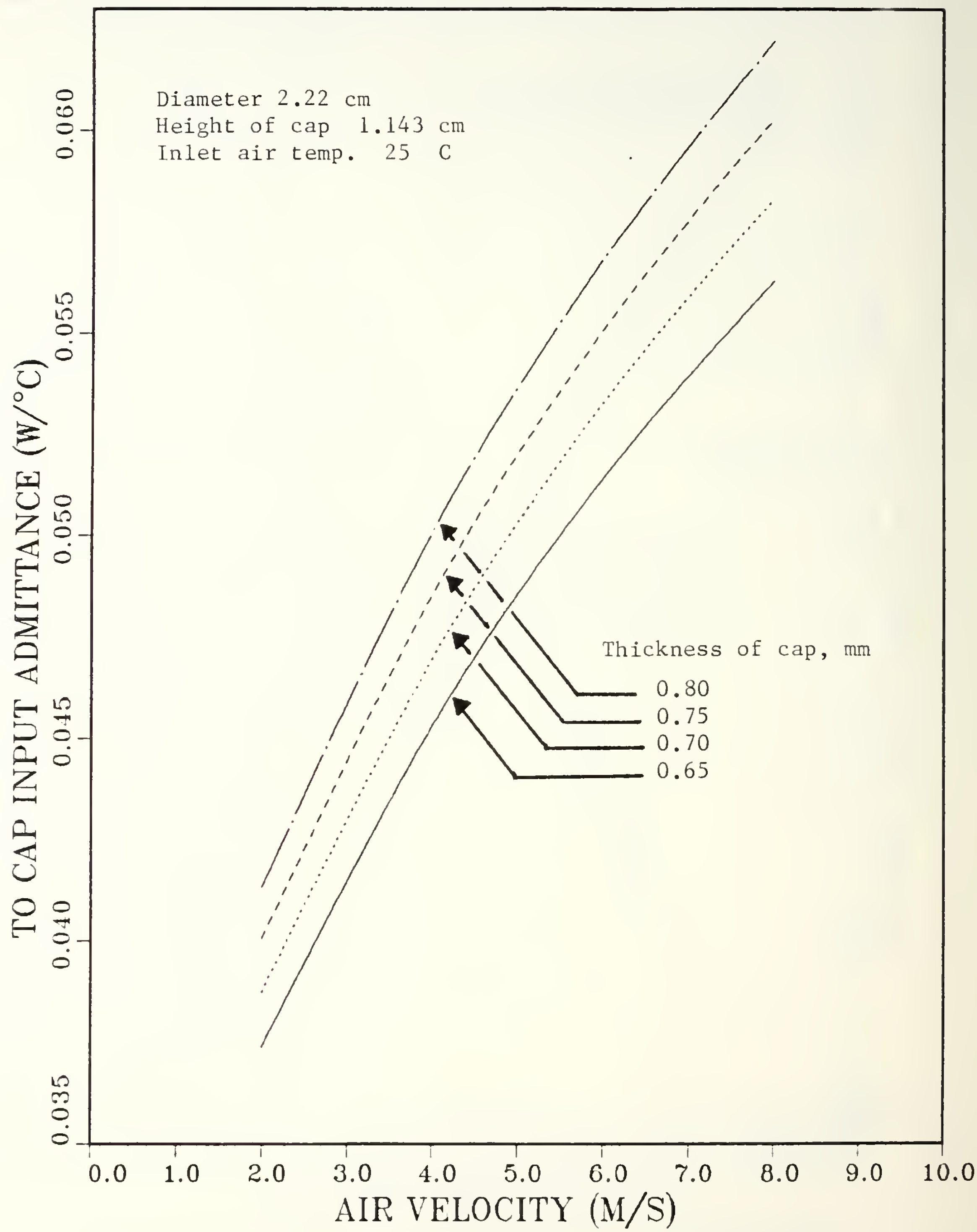

Figure 5.4 Input Admittance vs. Air Velocity with Varying Package Thickness.

VI. CONCLUSIONS

This thesis has provided a means to calculate the amount of heat that a transistor package will dissipate under operating conditions. The heat sinking system is the primary area where weight, volume and cost savings can be effected. It is extremely beneficial to the designer to have a precise estimate of the heat sinking system requirements. The small amount of heat dissipated by the transistor cap may nonetheless lead to smaller, less costly cooling structures. The savings here continue to translate down the line and the overall impact may be considerable.

Good engineering demands precision. Technology advances not purely through development of new systems, but also through continuous striving to obtain a better and deeper knowledge of existing systems and operating conditions. As a designer's knowledge of the capability of his available tools and equipment increases, the better he is able to provide effective and efficient products.

APPENDIX A

HEAT FLOW IN A RECTANGULAR FIN

1. ASSUMPTIONS

This appendix will review some basic heat transfer theory and develop equations that govern heat flow in a rectangular fin. The following assumptions, which are attributed to Murray [Ref. 12] and Gardner [Ref. 13], are made in these derivations:

1) Steady heat flow throughout the fin.

2) Heat transfer to or from the faces of the fin is proportional to the temperature difference between fin and surroundings. This eliminates radiation as a mode for this heat transfer.

3) There is no thermal resistance between the fin and the base surface.

4) Fin material is homogenous, with constant heat transfer coefficient and thermal conductivity.

5) No heat sources or sinks in the fin.

6) Temperature of surrounding medium is constant.

7) Temperature at base of fin is constant.

8) The dimensions of the fin are such that temperature gradients exist in the x direction only.

2. CONDUCTION

Heat flow through a body from an area of higher temperature to one of lower temperature occurs through the process of conduction. Heat flow by conduction is predicated to arise in two distinct ways: by molecular collisions with a resultant transfer of kinetic energy from a hotter to a cooler substance, and by electron drift. The kinetic energy transfer is commonly viewed as occurring between rapidly vibrating molecules of a substance and less rapidly vibrating adjacent molecules.

The amount of heat flow by conduction is directly proportional to the temperature gradient in the direction of the path of heat flow and to the area normal to the path of heat flow:

$$q = -A\, dT/dx \qquad (A.1)$$

where the minus sign indicates that if the temperature decreases with x, then q will be positive and will flow in the x direction. This relationship was first proposed by Joseph Fourier in 1822 [Ref. 4: p. 9]. An example of a heat flow path by conduction is shown in Figure A.1.

Insertion of a proportionality constant, k, results in the Fourier Law of Conduction:

$$q = -kA\, dT/dx \qquad (A.2)$$

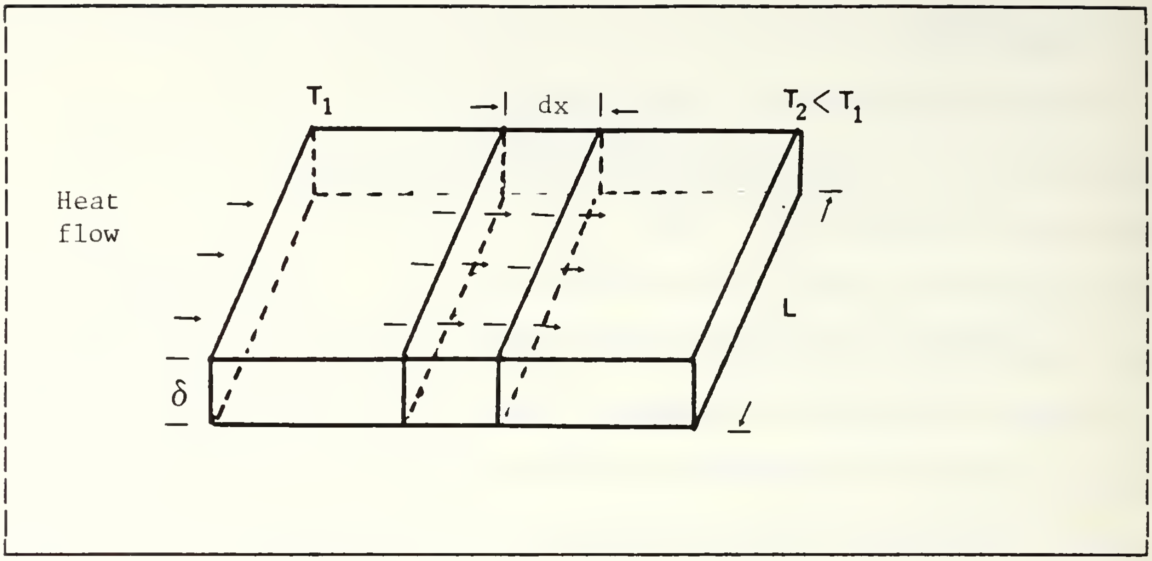

Figure A.1 Heat Flow Path by Conduction.

which serves to define the thermal conductivity of a particular material as:

$$k = (-q/A)/(dT/dx) \tag{A.3}$$

3. CONVECTION

Heat transfer at the interface between a solid and a fluid at different temperatures is significant in many common applications. When the surrounding fluid is completely stationary relative to the solid, heat transfer is purely by conduction. When flowing, however, the fluid forms a thin boundary layer around the solid into which heat is conducted [Ref. 4: p. 17]. This heat is then swept away with the removal of the fluid in a prevailing circulation

pattern. The transfer of heat in this fashion describes the convection mode of heat transfer.

The amount of heat transferred by convection is proportional to the surface area normal to the heat flow path and the difference in temperature between the surface and the bulk of the surrounding fluid:

$$q \propto S(T - T_a) = S\Theta \tag{A.4}$$

where T_a indicates the temperature of the fluid (or environment). Figure A.2 illustrates the flow of heat by convection.

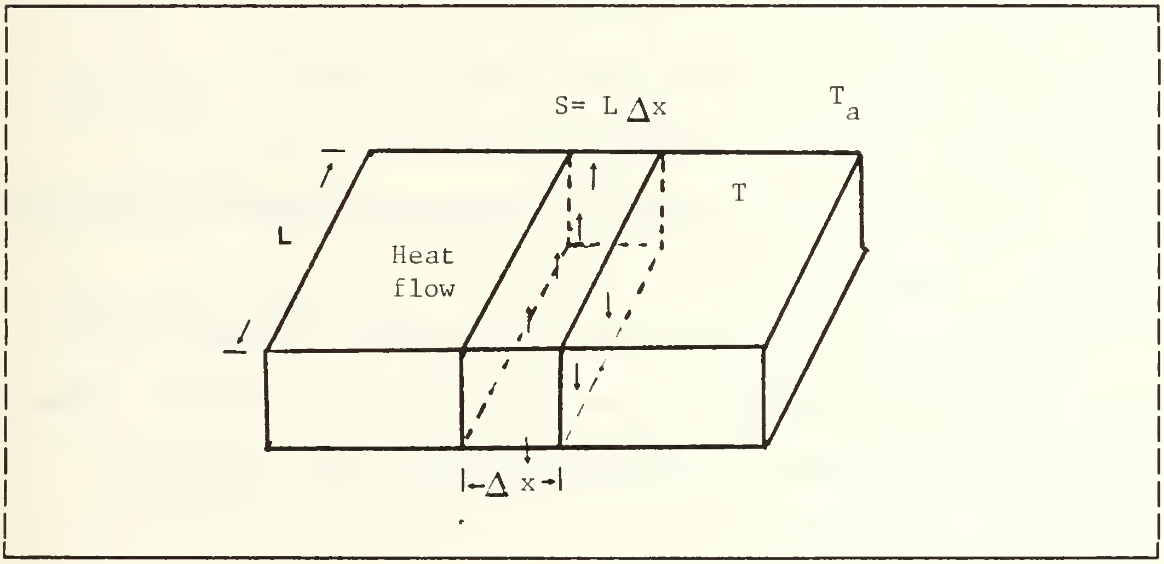

Figure A.2 Heat Flow Path by Convection.

The constant of proportionality used in convection, h, is the heat transfer coefficient. The provision of this

proportionality constant results in what is known as Newton's law of cooling by convection:

$$q = hS\Theta \tag{A.5}$$

and this serves to define h as:

$$h = q/(S\Theta) \tag{A.6}$$

The heat transfer coefficient may be viewed as a measure of the ease with which the convection process may proceed.

4. THE TEMPERATURE EXCESS AND HEAT FLOW IN THE COOLING FIN

The longitudinal fin of rectangular profile is shown with its terminology and coordinate system in Figure A.3. Note that the coordinate system has its origin at the fin tip and has a positive sense of direction toward the fin base. The slice in the middle indicates a differential element of width dx.

The principle of conservation of energy requires that the difference between the heat entering and leaving the differential element by conduction must equal the heat leaving by convection:

$$d/dx\ (kA\ dT/dx)dx = h(2Ldx)\ (T - T_a) \tag{A.7}$$

Assuming constant thermal conductivity and cross-sectional area, and defining the temperature excess Θ as:

$$\Theta = T - T_a \tag{A.8}$$

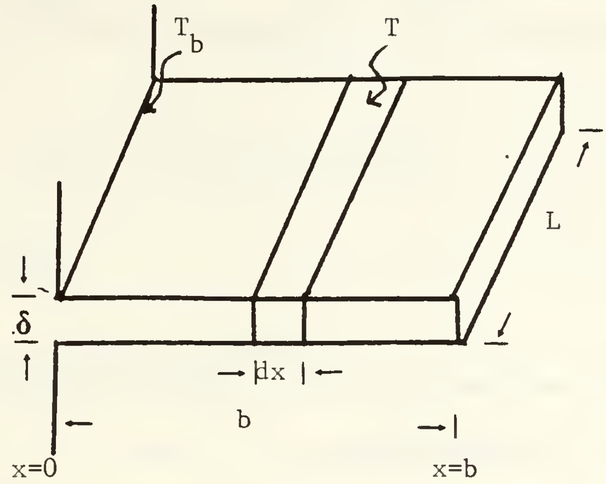

Figure A.3 Coordinate System of Cooling Fin.

so that

$$d\Theta = dT \tag{A.9}$$

equation A.7 then becomes

$$d^2\Theta/dx^2 - m^2\Theta = 0 \tag{A.10}$$

where

$$m = \sqrt{2h/k\delta} \tag{A.11}$$

The m term is regarded as the fin performance factor or fin "attenuation".

The general solution to equation A.10 is:

$$\Theta = C_1e^{mx} + C_2e^{-mx} \tag{A.12}$$

and a particular solution can be obtained in a "boundary value problem" based on the boundary conditions:

$$\Theta(x=b) = \Theta_b \tag{A.13}$$

$$q(x=0) = 0 \tag{A.14}$$

which makes the assumption that the heat flowing from the tip of the fin is negligible.

Substitution of these boundary conditions into equation A.12 gives:

$$\Theta(x=b) = \Theta_b = C_1 e^{mb} + C_2 e^{-mb} \tag{A.15}$$

Recognizing that the heat flow through the fin is in a direction opposite that of the positive sense of the coordinate system, the minus sign of equation A.2 may be eliminated, and it can be stated that:

$$q = kA\ dT/dx = kA\ d\Theta/dx = kAm\ (C_1 e^{mx} - C_2 e^{-mx}) \tag{A.16}$$

Then considering the heat flow at x = 0, it is seen that:

$$q(x=0) = 0 = C_1 - C_2 \tag{A.17}$$

which yields:

$$C_1 = C_2 \tag{A.18}$$

This result leads to:

$$C_1 = C_2 = \frac{\Theta_b}{e^{mb} + e^{-mb}} = \frac{\Theta_b}{2\cosh(mb)} \tag{A.19}$$

so that

$$\Theta(x) = \Theta_b \ (e^{mx} - e^{-mx})/2\cosh(mb) \tag{A.20}$$

or alternatively:

$$\Theta(x) = \Theta_b \cosh(mx)/\cosh(mb) \tag{A.21}$$

The heat flow in the fin will be:

$$q(x) = kA\, d\Theta/dx = kAm\Theta_b \sinh(mx)/\cosh(mb) \tag{A.22}$$

and at x=b, the heat entering the base of the fin is:

$$q(x{=}b) = kAm\Theta_b \sinh(mb)/\cosh(mb) \tag{A.23}$$

or

$$q(x{=}b) = kAm\Theta_b \tanh(mb) \tag{A.24}$$

With the cross-sectional area of the fin denoted as δL, then:

$$q(x{=}b) = Y_0\Theta_b \tanh(mb) \tag{A.25}$$

where Y_0 is defined to be:

$$Y_0 = L\sqrt{2hk\delta} \tag{A.26}$$

may be viewed as a sort of "characteristic admittance" of the fin.

APPENDIX B

HEAT TRANSFER COEFFICIENTS

This appendix lists the heat transfer coefficients used in the calculations of the curves in this thesis.

Heat Transfer Coefficients for the Top Disk

Air Velocity (m/s)	Inlet Air Temperature (°C)					
	20	25	30	40	50	70
1	17.488	17.470	17.449	17.405	17.364	17.291
2	24.968	24.932	24.894	24.813	24.739	24.604
3	30.881	30.829	30.774	30.660	30.553	30.361
4	35.990	35.921	35.850	35.703	35.566	35.318
5	40.587	40.503	40.415	40.236	40.068	39.765
6	44.824	44.723	44.619	44.408	44.211	43.853
7	48.788	48.671	48.551	48.308	48.081	47.669
8	52.538	52.404	52.268	51.994	51.737	51.272

Heat Transfer Coefficients for the Sides

Air velocity (m/s)	Inlet Air Temperature (°C)					
	20	25	30	40	50	70
1	21.080	21.101	21.499	21.070	21.034	21.044
2	29.122	29.141	29.144	29.101	29.055	29.068
3	35.613	35.486	35.321	35.150	35.097	35.112
4	42.536	42.390	42.194	41.747	41.324	40.691
5	48.825	48.659	48.434	47.920	47.435	46.707
6	54.649	54.464	54.210	53.633	53.092	52.278
7	60.103	59.911	59.629	58.210	58.399	57.503
8	65.289	65.051	64.758	64.070	63.422	62.450

APPENDIX C

COMPUTER PROGRAM FOR CALCULATING CURVE DATA

```
C *******************************************************************
C *   H1 = HEAT TRANSFER COEFFICIENT OF CYLINDER
C *   H2 = HEAT TRANSFER COEFFICIENT OF TOP DISK
C *   Y1 = CHARACTERISTIC ADMITTANCE OF CYLINDER
C *   Y2 = CHARACTERISTIC ADMITTANCE OF TOP DISK
C *   Z1 = CHARACTERISTIC IMPEDANCE OF CYLINDER
C *   K  = THERMAL CONDUCTIVITY OF CAP MATERIAL
C *   DELTA  = THICKNESS OF CAP MATERIAL
C *   B  = HEIGHT OF CYLINDER
C *   YIN = INPUT ADMITTANCE OF ENTIRE CAP
C *   TEMP = BULK AIR TEMPERATURE
C *   AIR = VELOCITY OF AIR FLOW
C *   RE = RADIUS OF CAP
C *******************************************************************


      REAL EMU,BESI0,BESI1,BESI2,BESK0,BESK1,BESK2,CAT,DOG,M,H,K,D,N
      REAL PI
C
C
C
      DO 5, I = 1,8
          READ (4,100) H2,H1
C         WRITE (6,200) I, H2, H1
C
C
          PI = 3.14159
          RE = 0.0222/2.0
          K = 16.0
          DELTA = .000762
          M =(H1/(K*DELTA))**.5
          N =(H2/(K*DELTA))**.5
          B = .020
          Y1 = 2*PI*RE*(H1*K*DELTA)**.5
          Y2 = 2*PI*RE*(H2*K*DELTA)**.5
C         WRITE (6,300) Y1,Y2,N
          Z1 = 1/Y1
C
C
          COSH = .5*(EXP(M*B)+EXP(-M*B))
          SINH = .5*(EXP(M*B)-EXP(-M*B))
C         WRITE (6,300) COSH,SINH
C
```

```
C
            CALL BESIK(N,BESIO,BESI1,BESI2,BESKO,BESK1,BESK2)
            BRATIO = BESI1/BESIO
C
C
C
            YTOP = Y1*SINH + Y2*COSH*BRATIO
            YDENOM = COSH + Z1*Y2*SINH*BRATIO
            YIN = YTOP/YDENOM

            WRITE(8,200) I, YIN
    5 CONTINUE
  100 FORMAT (2F10.6)
  200 FORMAT (1X,I2,8X,F10.6,10X,F10.6)
  300 FORMAT (1X,F15.10,5X,F15.10,5X,F15.10)
      END
C SUBROUTINE BESIK
C
C PURPOSE
C
C   COMPUTE THE MODIFIED BESSEL FUNCTIONS FOR A GIVEN ARGUMENT
C   AND FOR ORDERS FROM 0 TO 2.
C
C USAGE
C   CALL BESIK(EMU,BESIO,BESI1,BESI2,BESKO,BESK1,BESK2)
C
C DESCRIPTION OF PARAMETERS
C   EMU      -- THE ARGUMENT OF THE BESSEL FUNCTION DESIRED
C   BESIO    -- THE ZEROTH ORDER OF THE I BESSEL FUNCTION
```

```
C   BESI1    -- THE FIRST ORDER OF THE I BESSEL FUNCTION
C   BESI2    -- THE SECOND ORDER OF THE I BESSEL FUNCTION
C   BESK0    -- THE ZEROTH ORDER OF THE K BESSEL FUNCTION
C   BESK1    -- THE FIRST ORDER OF THE K BESSEL FUNCTION
C   BESK2    -- THE SECOND ORDER OF THE K BESSEL FUNCTION
C
C REMARKS
C
C   EMU MUST BE GREATER THAN OR EQUAL TO ZERO
C
C   NOTE:  SHOULD THE ARGUMENT OF THE SUBROUTINE BE EQUAL TO ZERO
C          AN ERROR COULD RESULT IN THE MAIN PROGRAM IF IT CONTAINS
C          EQUATIONS INVOLVING EXPONENTIAL OR LOG FUNCTIONS OF THE
C          ARGUMENT BESI1 OR BESI2.
C
C METHOD
C
C   COMPUTES FOR AN ORDER 0 AND 1 BOTH MODIFIED BESSEL FUNCTIONS USING
C   POLYNOMIAL APPROXIMATION.  FOR ORDER 2, A RECURRENCE RELATION IS
C   USED.
C
C   FOR POLYNOMIAL APPROXIMATION SEE STEGUN AND ABRAMAWITZ, AMS55
C
C**********************************************************************
C**********************************************************************
C
      SUBROUTINE BESIK(EMU,BESI0,BESI1,BESI2,BESK0,BESK1,BESK2)
C
C     FOR CALCULATION SIMPLICITY, TERMS FOR AN ARGUMENT OF ZERO ARE
C     DECLARED AS FOLLOWS:
C
      IF (EMU.EQ.0) GO TO 10
      GO TO 20
C
   10 BESI0 = 1.
      BESI1 = 0.
      BESI2 = 0.
      BESK0 = 9999.9999
      BESK1 = 9999.9999
      BESK2 = 9999.9999
      GO TO 999
```

```
C
C     CALCULATIONS FOR I(0),I(1),K(0),K(1) USING THE
C     POLYNOMIAL APPROXIMATION METHOD.
C
   20 CAT = EMU/3.75
      IF (EMU.GT.3.75) GO TO 30
C
      BESI0 = 1. + 3.5156229*(CAT**2.) + 3.0899424*(CAT**4.)
     1        + 1.2067492*(CAT**6.) + .2659732*(CAT**8.)
     2        + .0360768*(CAT**10.) + .0045813*(CAT**12.)
C
      BESI1 = EMU*(.5 + .87890594*(CAT**2.) + .51498869*(CAT**4.)
     1        + .15084934*(CAT**6.) + .02658733*(CAT**8.)
     2        + .00301532*(CAT**10.) + .00032411*(CAT**12.))
C
      IF (EMU.GT.2.) GO TO 40
C
      DOG = EMU/2.
C
      BESK0 = -ALOG(DOG)*BESI0 - .57721566 + .4227842*(DOG**2.)
     1        + .23069756*(DOG**4.) + .0348859*(DOG**6.)
     2        + .00262698*(DOG**8.) + .0001075*(DOG**10.)
     3        + .0000074*(DOG**12.)
C
      BESK1 = ALOG(DOG)*BESI1 + (1./EMU)*(1.+.15443144*(DOG**2.)
     1        - .67278579*(DOG**4.) - .18156897*(DOG**6.)
     2        - .01919402*(DOG**8.) - .00110404*(DOG**10.)
     3        - .00004686*(DOG**12.))
C
      GO TO 50
   30 BESI0 = SQRT(1./EMU)*EXP(EMU)*(.39894228 + .01328592/CAT
```

```
     1      + .00225319*(1./CAT**2.) - .00157565*(1./CAT**3.)          THEO1450
     2      + .00916281*(1./CAT**4.) - .02057706*(1./CAT**5.)          THEO1460
     3      + .02635537*(1./CAT**6.) - .01647633*(1./CAT**7.)          THEO1470
     4      + .00392377*(1./CAT**8.))                                  THEO1480
C                                                                      THEO1490
      BESI1 = SQRT(1./EMU)*EXP(EMU)*(.39894228 - .03988024*(1./CAT)    THEO1500
     1      - .00362018*(1./CAT**2.) + .00163801*(1./CAT**3.)          THEO1510
     2      - .01031555*(1./CAT**4.) + .02282967*(1./CAT**5.)          THEO1520
     3      - .02895312*(1./CAT**6.) + .01787654*(1./CAT**7.)          THEO1530
     4      - .00420059*(1./CAT**8.))                                  THEO1540
C                                                                      THEO1550
   40 DOG = EMU/2.                                                     THEO1560
C                                                                      THEO1570
      BESK0 = SQRT(1./EMU)*EXP(-EMU)*(1.25331414 - .07832358*(1./DOG)  THEO1580
     1      + .02189568*(1./DOG**2.) - .01062446*(1./DOG**3.)          THEO1590
     2      + .00587872*(1./DOG**4.) - .0025154*(1./DOG**5.)           THEO1600
     3      + .00053208*(1./DOG**6.))                                  THEO1610
C                                                                      THEO1620
      BESK1 = SQRT(1./EMU)*EXP(-EMU)*(1.25331414 + .23498619*(1./DOG)  THEO1630
     1      - .0365562*(1./DOG**2.) + .015042686*(1./DOG**3.)          THEO1640
     2      - .00780353*(1./DOG**4.) + .00325614*(1./DOG**5.)          THEO1650
     3      - .00068245*(1./DOG**6.))                                  THEO1660
C                                                                      THEO1670
C CALCULATIONS FOR I2(X),K2<X), USING RECURRENCE RELATIONS             THEO1680
C                                                                      THEO1690
   50 BESI2 = BESI0 -(2./EMU*BESI1)                                    THEO1700
      BESK2 = BESK0 + (2./EMU*BESK1)                                   THEO1710
C                                                                      THEO1720
  999 RETURN                                                           THEO1730
      END                                                              THEO1740
```

APPENDIX D

NOMENCLATURE

A	an arbitrary constant or Area, m
B	an arbitrary constant
C	an arbitrary constant, or capacitance, farads
D	an arbitrary constant
d	diameter, m
e	the Naperian base
G	Conductance, mhos
h	Heat Transfer Coefficient, W/m^2-s-°C
I	Current, amperes or Modified Bessel Function of First Kind
K	Modified Bessel Function of Second Kind
k	Thermal Conductivity, W/m-s-°C
L	Inductance, henries
l	length, m
m	Fin "attenuation" factor of fin or sides, 1/m
n	Fin "attenuation" factor of top, 1/m, or arbitrary constant
q	Heat flow, W
Pr	Prandtl number
R	Resistance, ohms
r	Radius, m
S	Surface area, m

T	Temperature, $^\circ$C
u	Free stream velocity, m/s
V	Voltage, volts
x	a coordinate
Y	Admittance, mhos in electrical case and W/$^\circ$C in thermal case
Z	Impedance, ohms in electrical case and $^\circ$C/W in thermal case
α	Attenuation constant, per unit length
β	Phase shift constant, per unit length
γ	Propagation constant, per unit length
Δ	change in variable
δ	Thickness of fin or transistor cap, m
ν	Kinematic viscosity (m^2/s)
θ	Temperature excess, $^\circ$C

Subscripts

a	refers to atmosphere or environment, or to end of fin
b	refers to base of fin
c	refers to center of the disk, or to the case
d	refers to the circular disk
e	refers to edge of disk
eq	refers to equivalent representation

j	refers to case junction
R	refers to receiving end of transmission line
S	refers to sending end of transmission line
s	refers to the side of the cap, or to free stream temperature
0	refers to characteristic impedance or admittance
1	refers to first two-port when cascaded
2	refers to second two-port when cascaded

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